DYNAMICS OF PARALLEL BOILING CHANNELS

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The stability of parallel-connected boiling channels is examined in the case of slight deviations from stationary equilibrium conditions. The conditions for stability are deduced and the influence of the parameters on the location of the stability limits is defined.

The stability of a system of boiling channels was examined in [1], where the pressure drops in the heated part of the parallel channels were considered negligible as compared with the pressure drops in the unheated parts.

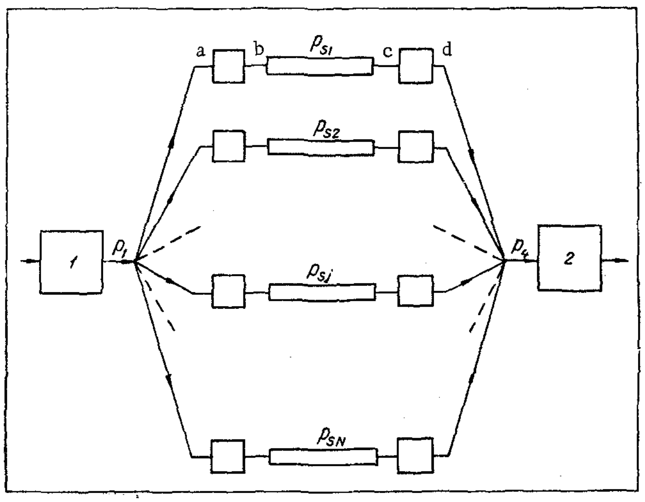

Fig. 1. Block diagram of the system of parallel boiling channels. (1, 2 are the external parts of the system.)

This paper deals with the stability under stationary conditions of a system (Fig. 1) consisting of parallel boiling channels and external (upstream and downstream) parts for any ratio of the pressure drops in the heated part of the channels (bc) and in the resistances (ab), (cd) lumped at the ends of the channels. The external parts (1, 2) of the system may represent pumps, pipes, or turbines. Pressure at the inlet and outlet of the system is assumed constant, while the heat flux supplied to each of the boiling channels is assumed sufficient to effect the complete evaporation of the liquid entering the channel.

To simplify the system of partial differential equations describing the process of heat and mass transfer in the boiling channels, the following assumptions were made:

1. The temperature of the external heater, and the density and specific heat of the liquid and the vapor are constant.

2. In the evaporating zone the volume of liquid is negligibly small in comparison with the volume of vapor. The mean velocity of the mixture in the evaporating zone is assumed to be equal to the velocity of the vapor at the outlet. (In the range of working conditions of practical interest, this assumption results in a certain reserve of stability.)

3. The heat transfer coefficient for the walls of the boiling channels is constant, and their specific heat is negligibly small.

The following boundary conditions were used:

1. The temperature of the liquid flowing into the inlet of the boiling channel is constant.

2. The pressure in the inlet header is given.

The boiling channels are assumed to be identical, i.e., the magnitudes of all the parameters characterizing the stationary regime are taken to be independent of the number j of the channel.

In the case of small deviations from the stationary equilibrium state, the equations of heat and mass transfer are solved in the same way as in [1]. With the above assumptions and boundary conditions, the solution of these equations (length of economizer zone and vapor velocity at outlet of boiling channel as functions of liquid velocity at channel inlet and saturation pressure) can be written as follows:

$$\frac{\overline{\Delta h_{e.j}}}{h_{e.o}} = \frac{1-\exp(-\lambda)}{\lambda}\,\frac{\overline{\Delta W_{in.j}}}{W_{in.o}} + \overline{\Delta P_{sj}}, \tag{1}$$

$$\frac{\overline{\Delta W_{out.j}}}{W_{out.o}} = [\sigma + (1-\sigma)\exp(-\lambda)]\,\frac{\overline{\Delta W_{in.j}}}{W_{in.o}} - (1-\sigma)\lambda\overline{\Delta P_{sj}}, \tag{2}$$

where $\sigma = \frac{\gamma_v}{\gamma_l}$, $\tau_e = \frac{s\gamma_l c_l}{k}\ln\frac{\vartheta_1-\vartheta_{in}}{\vartheta_1-\vartheta_{s.o}}$, $\lambda = z\tau_e$, P is the dimensionless pressure,

$$p = P\left(\frac{dp_s}{d\vartheta_s}\right)_o (\vartheta_1-\vartheta_{so})\ln\frac{\vartheta_1-\vartheta_{in}}{\vartheta_1-\vartheta_{so}}. \tag{3}$$

Neglecting the inertia properties of the external parts of the system, we can represent the relation between the pressure drop across these parts and the mass flow rate in the form of certain functions $p_1(G^*)$ and $p_4(G^*)$. Linearizing these relations and assuming turbulent self-similar flow conditions in the channel, we obtain

$$-\overline{\Delta P_1} = V_1\sum_{j=1}^{N}\overline{\Delta W_{in.j}}\Big/ NW_{in.o};$$

$$\overline{\Delta P_4} = V_2\sum_{j=1}^{N}\overline{\Delta W_{out.j}}\Big/ NW_{out.o}, \tag{4}$$

where

$$V_1 = -G_0^* \left(\frac{dP_1}{dG^*}\right)_0; \quad V_2 = G_0^* \left(\frac{dP_4}{dG^*}\right)_0; \quad G^* = \sum_{j=1}^{N} G_j.$$

The linearized equations of motion of liquid and vapor in the boiling channels are written thus:

$$\overline{\Delta P_1} - \overline{\Delta P_4} = (U_1 + U_2)\frac{\overline{\Delta W_{\text{in}.j}}}{W_{\text{in.o}}} + (U_3 + U_4)\frac{\overline{\Delta W_{\text{out}.j}}}{W_{\text{out.o}}} + \left(\frac{1-\sigma}{\sigma}\right) U_2 \varepsilon \frac{\overline{\Delta h_{e.j}}}{h_{e.o}};$$

$$\overline{\Delta P_1} - \overline{\Delta P_{sj}} = (U_1 + U_2)\frac{\overline{\Delta W_{\text{in}.j}}}{W_{\text{in.o}}} + U_2 \varepsilon \frac{\overline{\Delta h_{e.j}}}{h_{e.o}};$$

$$U_1 = 2(P_1 - P_2)_0; \quad \varepsilon = \xi(h_{e.o}) h_{e.o} \Big/ 2\int_0^{h_{e.o}} \xi(x)\,dx;$$

$$U_2 = 2(P_2 - P_5)_0;$$

$$U_3 = 2(P_5 - P_3)_0; \quad U_4 = 2(P_3 - P_4)_0. \tag{5}$$

Then the characteristic equation of the system, obtained from the condition that there exist a nontrivial solution to Eqs. (1)–(5), has the following form (F_1 defines the stability of the system in the small with respect to interchannel pulsation, F_2 with respect to pulsation in the boiler as a whole):

$$(F_1)^{N-1} \cdot F_2 = 0;$$

$$F_n = -A_n[1 - \exp(-\lambda)]/\lambda + B_n\lambda + C_n \exp(-\lambda) + 1;$$

$$A_n = (1-\sigma)\Phi_{1n}/(\Phi_{1n} + \sigma m_n)(\Phi_{1n} + \sigma\Phi_{2n}); \quad B_n = A_n m_n \sigma \Phi_{2n};$$

$$C_n = A_n \sigma m_n \Phi_{2n}/\Phi_{1n}; \quad m_n = \Phi_{1n}/\varepsilon U_2; \quad n = 1,\ 2;$$

$$\Phi_{11} = U_1 + U_2; \quad \Phi_{12} = \Phi_{11} + V_1;$$

$$\Phi_{21} = U_3 + U_4; \quad \Phi_{22} = \Phi_{21} + V_2. \tag{6}$$

Thus, the stability analysis reduces to a study of the roots of the equation

$$-A[1 - \exp(-\lambda)]/\lambda + B\lambda + C\exp(-\lambda) + 1 = 0. \tag{7}$$

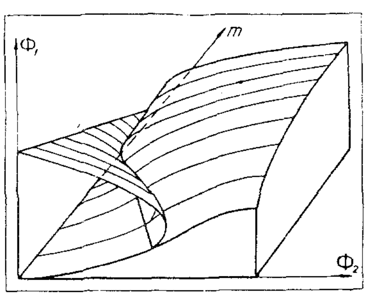

Fig. 2. Qualitative picture of the D-partition of the space of the parameters Φ_1, Φ_2, m.

Carrying out a D-partition of the space of the parameters A, B, C, we find that the characteristic equation (7) does not have roots with a positive real part if parameters A, B, C belong to the region M bounded by the surface of the D-partition

$$A = \frac{\omega(\omega B\cos\omega + \sin\omega)}{1 + \cos\omega},$$

$$C = 1 + \omega B\frac{\sin\omega}{1 - \cos\omega}, \quad 0 \leqslant \omega \leqslant 2\pi \tag{8}$$

and by the singular planes

$$A = 1 + C, \quad B = 0. \tag{9}$$

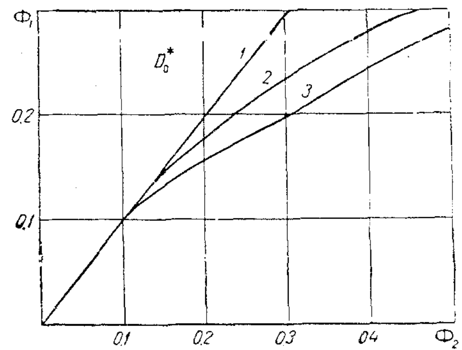

Fig. 3. Influence of the parameter σ on the location of the boundary of the stability region D_0^*: 1) with $\Phi_1 = \Phi_2$, Petrov's criterion; with $\sigma = 0$; 3) $\sigma = 0.079$.

The question of which of the sides of the bounding surfaces is external to the region M is determined by the hatching rule [2].

It is necessary to note that the stability region obtained cannot be fully realized in the system investigated, because the assumptions employed restrict the set of permissible values of the parameters. These restrictions reduce to satisfaction of the condition

$$\left(\frac{H - h_e}{h_e}\right)_0 \frac{c_l(\vartheta_1 - \vartheta_{so})}{r} \ln\frac{\vartheta_1 - \vartheta_{\text{in}}}{\vartheta_1 - \vartheta_{so}} > 1.$$

Since the parameters Φ_1, Φ_2, m are more convenient for engineering calculations, it is expedient to map the stability region M into the space Φ_1, Φ_2, m at a fixed value of $\sigma \in (0, 1)$. From (6) we obtain the following conversion formulas:

$$m^2 + m[A \cdot B + \sigma C^2 - (1-\sigma)C]/AC\sigma + B/A\sigma = 0;$$

$$\Phi_1 = B/C, \quad \Phi_2 = B/mA\sigma. \tag{10}$$

We shall consider only physically realizable parameters,

$$\Phi_1 > 0, \quad \Phi_2 > 0. \tag{11}$$

Then the boundary surface of the D-partition of the space of the parameters Φ_1, Φ_2, m, a qualitative picture of which is given in Fig. 2, is determined from (8), (10) with $0 \le \omega \le \pi$. For a given m each section of the space Φ_1, Φ_2, m, constitutes a mapping of the surface of intersection of the space A, B, C with the ruled surface defined by the equation

$$B = mC[(1-\sigma)/(C + mA) - \sigma]. \tag{12}$$

As follows from (8), (9), (11), and (12), there exists a value $m^* = (1-\sigma)/\sigma$ such that for $m < m^*$ the stability region is bounded both by the surface of the

D-partition and by the singular surface $\omega = 0$ (when $m < m^*$ both aperiodic and oscillatory instability are possible), whereas for $m > m^*$ the stability region is bounded only by the surface of the D-partition (when $m > m^*$ only oscillatory instability can occur).

It will be shown that aperiodic instability will be present in the system if and only if the operating point (the point characterizing the stationary regime) lies on the drooping branch of the static hydraulic characteristic of the boiling channel.

Indeed, the equation of the static hydraulic characteristic can be written in the form

$$\Phi = f(G, P_S), \text{ where } \Phi = \Phi_1 + \Phi_2.$$

Then the condition for the absence of a drooping branch is written

$$\frac{d\Phi}{dG} = \frac{\partial \Phi}{\partial G} + \frac{\partial \Phi}{\partial P_s}\frac{dP_s}{dG} \geqslant 0. \tag{13}$$

Inequality (16) can be rewritten thus:

$$\Phi + \frac{U_2}{\sigma}(\Phi_1 + \sigma\Phi_2 + \sigma - 1) \geqslant 0. \tag{14}$$

At the same time, (14) coincides with the condition requiring [as follows from (7)–(9)] that the departure of the system from the stability region not be accompanied by the appearance of a zero root (aperiodic instability).

Thus, aperiodic instability is possible only when the operating point is located on the drooping branch of the static hydraulic characteristic of the channel.

It follows from Eqs. (8)–(12) that the cylindrical surface $\Phi_1^* = \Phi_1(\Phi_2, m)$, whose generator is parallel to the m axis, while its directrix coincides with the boundary of the stability region at $m = \infty$ ($\varepsilon = 0$), lies above (Fig. 2) the boundary surface of the D-partition $\Phi_1 = \Phi_1(\Phi_2,\ m < +\infty,\ \omega \neq 0)$. Therefore to exclude the possibility of oscillatory instability in the system it is sufficient for the parameters Φ_1 and Φ_2 to belong to the region D_0^*, i.e.,

$$(\Phi_1,\ \Phi_2) \in D_o^*, \tag{15}$$

located in the first quadrant of the plane Φ_2, Φ_1 and above the curve $\Phi_1 = \Phi_1(\Phi_2, m = \infty)$ defined by the following equations:

$$\Phi_1 = \sin\omega/\omega,$$

$$\Phi_2 = -\sin\omega/\omega\,[\sigma + (1-\sigma)\cos\omega],$$

$$\frac{\pi}{2} + \arccos\frac{\sigma}{1-\sigma} \leqslant \omega \leqslant \pi. \tag{16}$$

From (16) it follows that the location of the boundary of the region D_0^* depends only on the one parameter σ, which is usually much less than unity. (It also follows from (16) that the region D_0^* will occupy the entire first quadrant of the plane Φ_2, Φ_1, if $\sigma \geq 1/2$.) The magnitude of the parameter σ has a major influence on the location of the stability limit only when ω is close to $\pi/2$, i.e., when $\Phi_2 \gg 1$ (Fig. 3). For this reason, in most cases of practical importance ($\Phi_2 < 1$, $\omega \approx \pi$), instead of (16) it is possible to use a much simpler relation of the form

$$\Phi_1 = \sin\omega/\omega,\quad \Phi_2 = -\operatorname{tg}\omega/\omega,$$

$$\pi/2 \leqslant \omega \leqslant \pi, \tag{17}$$

which is nevertheless sufficiently close to (16).

It will be noted that as $\omega \to \pi$, the condition determining the stability of the system in accordance with formula (17) goes over into the familiar criterion of P. A. Petrov [3].

The following conclusions may be drawn on the basis of an analysis of the roots of the characteristic equation (17):

1. The region of stability of the system grows with increase in the pressure in the boiling channel and the resistances lumped in the inlet part of the system and at the inlet to the boiling channel, and with decrease in the heat transfer coefficient, subcooling of the liquid at the channel inlet, and the resistances lumped at the outlet from the boiling channel and in the outlet part of the system.

2. The period of the oscillations occurring in the system on departure from the stability region across the boundary D-curve is of the same order of magnitude as the time taken by the liquid to pass through the economizer zone.

Thus, to ensure stability of the stationary regime in a system consisting of parallel-connected boiling channels and external parts, it is sufficient if the following conditions are satisfied:

1. The points determining the stationary regime of the system as a whole and of the individual channels must be located on the ascending branches of the corresponding static hydraulic characteristics.

2. The parameters Φ_1 and Φ_2 must belong to a region D_0^* located in the first quadrant of the plane (Φ_2, Φ_1) and above the curve $\Phi_1 = \Phi_1(\Phi_2, m = \infty)$, defined by formula (16).

In conclusion, the authors would like to take the opportunity to express their thanks to E. F. Sabayev for his valued advice.

NOTATION

p_1, p_4—pressures in the inlet and outlet headers, resp.; p_2—pressure beyond the resistance concentrated at the inlet end of the boiling channel; p_s—saturation pressure; p_3—pressure ahead of the resistance concentrated at the outlet end of the boiling channel; G—mass flow rate; W—velocity; γ—density; k—heat transfer coefficient; c—specific heat; ϑ—temperature; ϑ_1 and ϑ_s temperature of external heater and on saturation line; r—latent heat of evaporation; s—area of channel cross section; ξ—hydraulic friction coefficient; H—length of heated channel section; h_e—length of economizer zone; N—number of boiling channels; z—Laplace transformation parameter. Subscripts: in—inlet; out—outlet; l—liquid; v—vapor; o—stationary value in the neighborhood of which linearization is performed; Δ—deviation of variable from its stationary value; Laplace transforms of variables are denoted by a bar ($\overline{\Delta W}$—transform of ΔW).

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